

Edexcel Core 2 Jan 2008

1. (a) Find the remainder when

$$x^3 - 2x^2 - 4x + 8$$

is divided by (i) $x - 3$, (ii) $x + 2$. (3)

- (b) Hence, or otherwise, find all the solutions to the equation

$$x^3 - 2x^2 - 4x + 8 = 0. \quad (4)$$

2. The fourth term of a geometric series is 10 and the seventh term of the series is 80.

For this series, find

(a) the common ratio, (2) (b) the first term, (2)

- (c) the sum of the first 20 terms, giving your answer to the nearest whole number. (2)
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3. (a) Find the first 4 terms of the expansion of $\left(1 + \frac{x}{2}\right)^{10}$ in ascending powers of x , giving each term in its simplest form. (4)

- (b) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places. (3)
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4. (a) Show that the equation

$$3 \sin^2 \theta - 2 \cos^2 \theta = 1$$

can be written as

$$5 \sin^2 \theta = 3.$$

(2)

- (b) Hence solve, for $0^\circ \leq \theta < 360^\circ$, the equation

$$3 \sin^2 \theta - 2 \cos^2 \theta = 1,$$

giving your answers to 1 decimal place.

(7)

5. Given that a and b are positive constants, solve the simultaneous equations

$$a = 3b,$$

$$\log_3 a + \log_3 b = 2.$$

Give your answers as exact numbers.

(6)

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6.

Figure 1

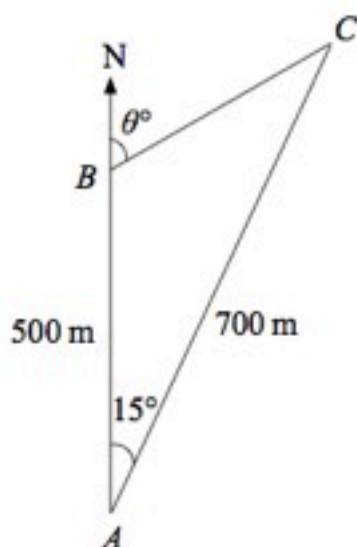


Figure 1 shows 3 yachts A , B and C which are assumed to be in the same horizontal plane. Yacht B is 500 m due north of yacht A and yacht C is 700 m from A . The bearing of C from A is 015° .

- (a) Calculate the distance between yacht B and yacht C , in metres to 3 significant figures.

(3)

The bearing of yacht C from yacht B is θ° , as shown in Figure 1.

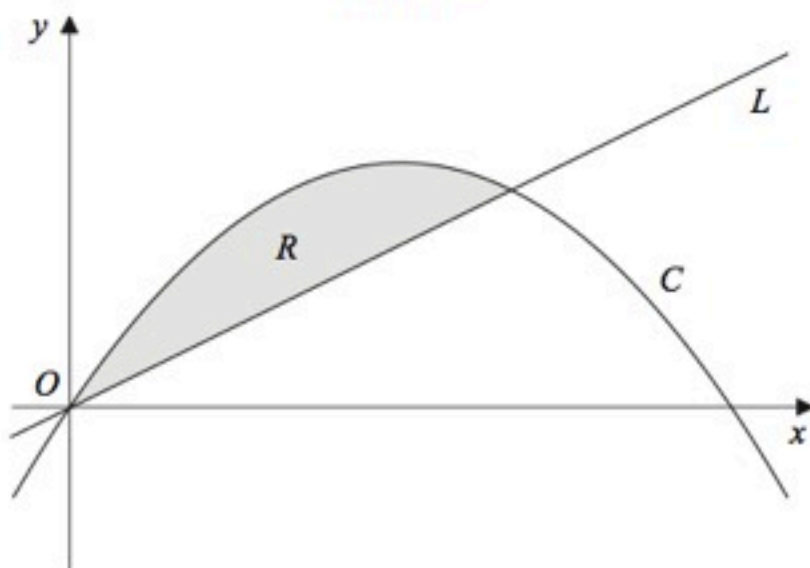
- (b) Calculate the value of θ .

(4)

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7.

Figure 2



In Figure 2 the curve C has equation $y = 6x - x^2$ and the line L has equation $y = 2x$.

(a) Show that the curve C intersects the x -axis at $x = 0$ and $x = 6$. (1)

(b) Show that the line L intersects the curve C at the points $(0, 0)$ and $(4, 8)$. (3)

The region R , bounded by the curve C and the line L , is shown shaded in Figure 2.

(c) Use calculus to find the area of R . (6)

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8. A circle C has centre $M(6, 4)$ and radius 3.

(a) Write down the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = r^2.$$

(2)

Figure 3

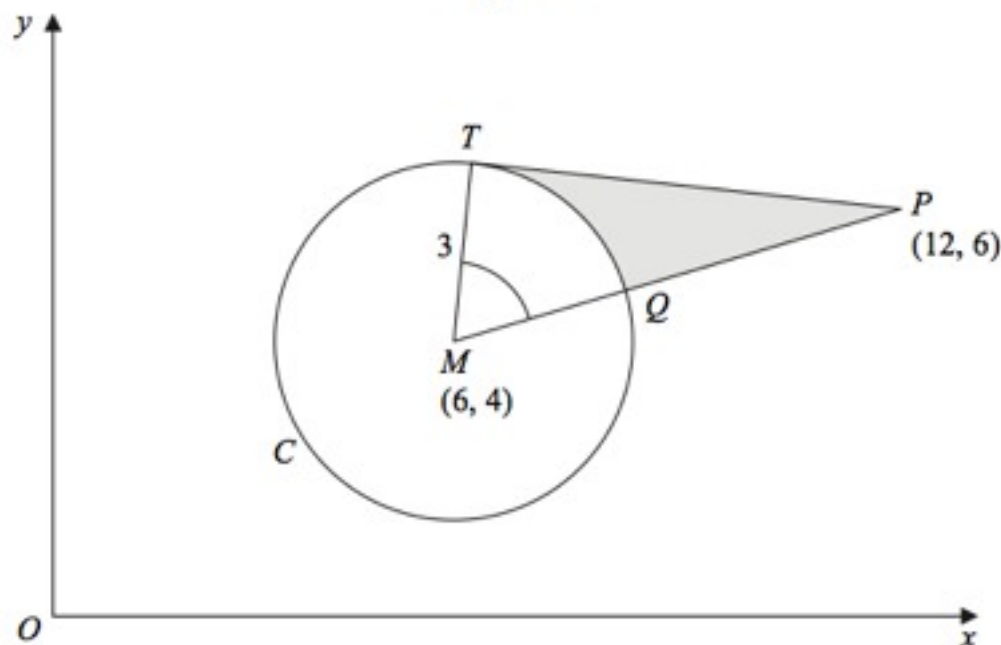


Figure 3 shows the circle C . The point T lies on the circle and the tangent at T passes through the point $P(12, 6)$. The line MP cuts the circle at Q .

(b) Show that the angle TMQ is 1.0766 radians to 4 decimal places.

(4)

The shaded region TPQ is bounded by the straight lines TP , QP and the arc TQ , as shown in Figure 3.

(c) Find the area of the shaded region TPQ . Give your answer to 3 decimal places.

(5)

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9.

Figure 4

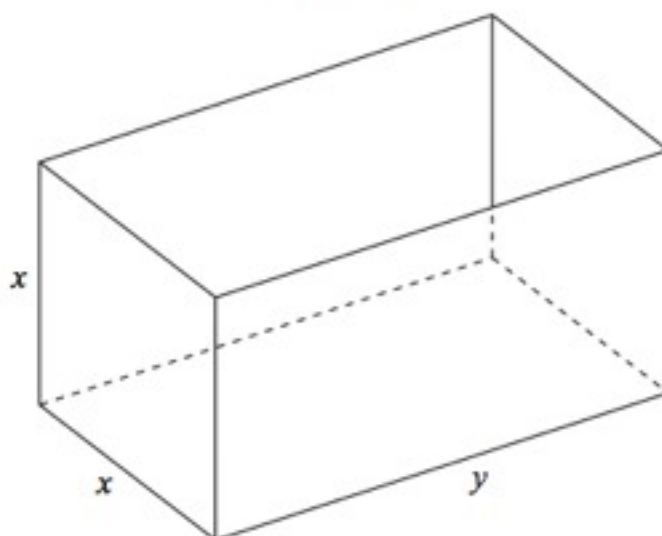


Figure 4 shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.

The capacity of the tank is 100 m^3 .

- (a) Show that the area $A \text{ m}^2$ of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2. \quad (4)$$

- (b) Use calculus to find the value of x for which A is stationary. (4)
- (c) Prove that this value of x gives a minimum value of A . (2)
- (d) Calculate the minimum area of sheet metal needed to make the tank. (2)
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